INDUS RANGERS DEFENCE ACADEMY

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NDA [WEEKLY TEST]
PRACTICE SHEET – 2

General Instructions:

- There are 60 questions
- All questions have 4 marks for correct and –1 mark for wrong answer as a penalty.
- The name, date, and exam must be filled in OMR sheet otherwise the result will not be published.
- OMR fill should be properly.

1. Under what conditions is the tangent to a given curve at a point perpendicular to x – axis?
   (a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 0$ (d) $\frac{d^2y}{d^2x} = 1$
2. If f(x) = (x–x_0) ϕ(x) and ϕ(x) is continuous at x = x_0, then what is f'(x_0) equal to?
   (a) $\phi'(x_0)$ (b) $\phi(x_0)$ (c) $x_0\ \phi(x_0)$ (d) $2\phi\ (x_0)$
3. The sum of two numbers is 20. What are the numbers if the product of the square of one and the cube of the order is maximum?
   (a) 6, 14 (b) 15, 5 (c) 12, 8 (d) 10, 10
4. What is the slope of the normal at point (at^2, 2at) of the parabola $y^2 = 4ax$?
   (a) 1/ t (b) t (c) – t (d) $-\frac{1}{t}$
5. Which one of the following statements is not correct?
   (a) The derivatives of f(x) at x = a is the slope of the graph of f(x) at the point [a, f(a)]
   (b) f(x) has a positive derivative at x = a means f(x) increases as x increases from 'a'
   (c) The sum of two differentiable functions is differentiable
   (d) If a function is continuous at a point, it is also differentiable at the same point.
6. Which one of the following statements is correct in respect of the curve $4y - x^2 - 8 = 0$?
   (a) The curve is increasing in (– 4, 4)
   (b) The curve is increasing in (– 4, 0)
   (c) The curve is increasing in (0, 4)
   (d) The curve is decreasing in (– 4, 4)
7. What is the minimum value of px + qy (p > 0, q > 0) when $xy = r^2$?
   (a) $2r\sqrt{pq}$ (b) $2pq\sqrt{r}$ (c) $-2r\sqrt{pq}$ (d) $2\,rpq$
8. What is/are the critical point(s) of the function $f(x) = x^{2/3}(5 – 2x)$ on the interval [– 1, 2]?
   (a) 1 only (b) 0, 1 (c) $\frac{3}{2}$ only (d) 0, $\frac{3}{2}$
9. If x + y = 12, what is the maximum value of xy?
   (a) 25 (b) 36 (c) 49 (d) 64

10. What is the x – coordinate of the point on the curve $f(x) = \sqrt{x}(7x - 6)$, where the tangent is parallel to x – axis?
    (a) $-\frac{1}{3}$ (b) $\frac{2}{7}$ (c) $\frac{6}{7}$ (d) $\frac{1}{2}$
11. What is the interval in which the function $f(x) = \sqrt{9 - x^2}$ is increasing? (f(x) > 0).
    (a) 0 < x < 3 (b) – 3 < x < 0
    (c) 0 < x < 9 (d) – 3 < x < 3
12. A wire 34 cm long is to be bent in the form of a quadrilateral of which each angle is 90°. What is the maximum area which can be enclosed inside the quadrilateral?
    (a) 68 cm^2 (b) 70 cm^2 (c) 71.25 cm^2 (d) 72.25 cm^2
13. The motion of a particle is described as $s = 2 – 3t + 4t^3$. What is the acceleration of the particle at the point where its velocity is zero?
    (a) 0 (b) 4 unit (c) 8 unit (d) 12 unit
14. What is the product of two parts of 20, such that the product of one part and the cube of the other is maximum?
    (a) 75 (b) 91 (c) 84 (d) 96
15. What is the maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$?
    (a) 1 (b) 2 (c) 5 (d) – 23
16. What is the area of the largest rectangular field which can be enclosed with 200 m of fencing?
    (a) 1600 m^2 (b) 2100 m^2
    (c) 2400 m^2 (d) 2500 m^2
17. What is the smallest value of m for which $f(x) = x^2 + mx + 5$ is increasing in the interval $1 \leq x \leq 2$?
    (a) m = 0 (b) m = – 1
    (c) m = – 2 (d) m = – 3
18. What is the maximum value of x . y subject to the condition x + y = 8?
    (a) 8 (b) 16 (c) 24 (d) 32
19. What is the equation of the curve whose slope at any point is equal to 2x and which passes through the origin?
    (a) $y(1 - x) = x^2$ (b) $y^2(1 - x^2) = x^4$
    (c) $y^2 = (x + 1)^2$ (d) $y = x^2$
20. What is the maximum value of the function log x – x?
    (a) – 1 (b) 0 (c) 1 (d) ∞
21. $f(x) = \cos x$ is monotonic decreasing under which one on the following conditions?
    (a) $0 < x < \frac{\pi}{2}$ only (b) $\frac{\pi}{2} < x\ \pi$ only
    (c) $0 < x < \pi$ (d) $0 < x < 2\pi$
22. What is the minimum value of $2x^2 – 3x + 5$?
    (a) 0 (b) ¾ (c) 31/4 (d) 31/8

23. A stone thrown vertically upward satisfies the equation s = 64t – 16t^{2,} where s is in meter and t is in second. What is the time required to reach the maximum height?
    (a) 1s (b) 2s (c) 3s (d) 4s
24. If $x\cos\theta + y\sin\theta = 2$ is perpendicular to the line x – y = 3, then what is one of the value of θ?
    (a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) $\pi/3$
25. The function $y = \tan^{-1}x - x$
    (a) is always decreasing (b) is always increasing
    (c) first increases and then decreases
    (d) first decreases and then increases
26. The velocity of telegraphic communication is given by $v = x^2\log(1/x)$, where x is the displacement. For maximum velocity, x equals to?
    (a) $e^{1/2}$ (b) $e^{-1/2}$ (c) $(2e)^{-1}$ (d) $(2e)^{-1/2}$
27. What is the maximum point on the curve x = e^xy?
    (a) (1, e) (b) (1, e^{-1}) (c) (e, 1) (d) (e^{-1}, 1)
28. A balloon is pumped at the rate of 4cm^3 per second. What is rate at which its surface area increases and radius is 4 cm?
    (a) 1 cm^2/s (b) 2 cm^2/s (c) 3 cm^2/s (d) 4 cm^2/s
29. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then which one of the following is correct?
    (a) k < 3 (b) k ≤ 3 (c) k > 3 (d) k ≥ 3
30. Given two squares of sides x and y such that y = x + x^2. What is the rate of change of area of the second square with respect to the area of the first square?
    (a) 1 + 3x + 2x^2 (b) 1 + 2x + 3x^2
    (c) 1 – 2x + 3x^2 (d) 1 – 2x – 3x^2
31. What is $\lim_{x\to 2}\frac{2-x}{x^3-8}$ equal to?
    (a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{12}$ (d) $-\frac{1}{12}$
32. A function $f : R \to R$ is defined as $f(x) = x^2$ for x ≥ 0 and $f(x) = -x$ for x < 0. Consider the following statements in respect of the above function:
    A. The function is continuous at x = 0.
    B. The function is differentiable at x = 0.
    Which of the above statements is/are correct?
    (a) A only (b) B only
    (c) Both A and B (d) Neither 1 nor 2
33. What is $\lim_{x\to 0}\frac{1-\cos x}{x}$ equal to?
    (a) 0 (b) $1/2$ (c) 1 (d) 2
34. What is $\lim_{x\to 0}\frac{\cos x}{\pi - x}$ equal to?
    (a) 0 (b) p (c) $\frac{1}{\pi}$ (d) 1

35. What is $\lim_{x\to 0} \frac{\sin 2x+4x}{2x+\sin 4x}$ equal to?

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

36. Let N denote the set of all non-negative integers and Z denote the set of all integers. The function $f : Z \to N$ given by f(x) = |x| is:

(a) One – one but not onto

(b) Onto but not one – one

(c) Both one – one and onto

(d) Neither one – one nor onto

37. What is $\lim_{x\to 0} \frac{(1+x)^n-1}{x}$ equal to?

(a) 0 (b) 1 (c) n (d) n – 1

38. What is $\lim_{x\to 0} \frac{x}{\sqrt{1-\cos x}}$ equal to?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) Limit does not exist

Direction (Qs. 39–40): For the next two (02) items that follow:

Consider the function $f(x) = \frac{1-\sin x}{(\pi-2x)^2}$, where $x \neq \frac{\pi}{2}$ *and* $f\left(\frac{\pi}{2}\right) = \lambda$.

39. What is $\lim_{x\to \frac{\pi}{2}} f(x)$ equal to?

(a) 1 (b) ½

(c) ¼ (d) 1/8

40. What is the value of λ if the function is continuous at x = π/2?

(a) 1/8 (b) ¼ (c) ½ (d) 1

41. If f (9) = 9 and f '(9) = 4 then what is $\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ equal to?

(a) 36 (b) 9

(c) 4 (d) None of these

42. Consider the following statements:

A. The function $f(x) = \sqrt[3]{x}$ is continuous at all x except at x = 0.

B. The function f(x) = [x] is continuous at x = 2.99 where [.] is the bracket function.

Which is the above statements is/are correct?

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

43. Consider the following statements:

A. The function $f(x) = |x|$ is not differentiable at x = 1.

B. The function f(x) = e^x is not differentiable at x = 0. Which of the above statements is/are correct?

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

Directions (Qs. 44–46): For the next three (03) items that follow:

Let f(x) be a function defined in $1 \leq x < \infty$ by

$$f(x) = \begin{cases} 2 - x, & for\ 1 \leq x \leq 2 \\ 3x - x^2, & for\ x > 2 \end{cases}$$

44. Consider the following statements:
    A. The function is continuous at every point in the interval (1, ∞).
    B. The function is differentiable at x = 1.5.
    Which of the above statements is/are correct?
    (a) A only (b) B only
    (c) Both A and B (d) Neither A nor B
45. What is the differentiable coefficient of f(x) at x = 3.
    (a) 1 (b) 2 (c) – 1 (d) – 3
46. Consider the following statements:
    A. f '(2 + 0) does not exist.
    B. f '(2 – 0) does not exist
    Which of the above statements is/are correct?
    (a) A only (b) B only
    (c) Both A and B (d) Neither A nor B

Directions (Qs. 47–49): For the next three (03) items that follow:

Consider the function $\boldsymbol{f(x) = \frac{x-1}{x+1}}$.

47. What $= \boldsymbol{\frac{f(x)+1}{f(x)-1} + x}$ is equal to?
    (a) 0 (b) 1 (c) 2x (d) 4x
48. What is f(2x) equal to?
    (a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{f(x)+1}{3f(x)+1}$
    (c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x)+3}{3f(x)+1}$
49. What is f(f(x)) equal to?
    (a) x (b) – x
    (c) – 1/x (d) None of these

Directions (Qs. 50 – 52): For the next three (03) items that follow:

Consider the function $f(x) = \begin{cases} x^2 - 5, & x \leq 3 \\ \sqrt{x + 13}, & x > 3 \end{cases}$

50. What is $\lim_{x \to 3} f(x)$ equal to?
    (a) 2 (b) 4
    (c) 5 (d) 13
51. Consider the following statements:
    A. The function is discontinuous at x = 3.
    B. The function is not differentiable at x = 0.

What of the above statements is/are correct?

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

52. What is the differentiable coefficients of f(x) at x = 12?

(a) 5/2 (b) 5 (c) 1/5 (d) 1/10

53. Consider the function $f(x) = \begin{cases} \frac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases}$

What is the non-zero value of k for which the function is continuous at x = 0?

(a) ¼ (b) ½ (c) 1 (d) 2

54. Consider the following statements:

A. The function f(x) = [x], where [.] is the greatest integer function defined on R, is continuous at all points except at x = 0.

B. The function f(x) = sin |x| is continuous for all $x \in R$.

Which of the above statements is/are correct?

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

55. What is $\lim_{x \to 0} \frac{\log_5(1+x)}{x}$ equal to?

(a) 1 (b) $\log_5 e$ (c) $\log_e 5$ (d) 5

56. What is $\lim_{x \to 0} \frac{5^x - 1}{x}$ equal to?

(a) $\log_e 5$ (b) $\log_5 e$ (c) 5 (d) 1

57. What is $\lim_{x \to \infty} \frac{1+2+3+\cdots+n}{1^2+2^2+3^3+\cdots n^2}$ equal to?

(a) 5 (b) 2 (c) 1 (d) 0

Directions (Qs. 58 – 59): For the next two (02) items that follow:

Given that $\lim_{x \to \infty} \left(\frac{2+x^2}{1+x} - Ax - B \right) = 3.$

58. What is the value of A?

(a) – 1 (b) 1 (c) 2 (d) 3

59. What is the value of B?

(a) – 1 (b) 3 (c) – 4 (d) – 3

60. If $G(x) = \sqrt{25 - x^2}$, then what is $\lim_{x \to 1} \frac{G(x) - G(1)}{x - 1}$ equal to?

(a) $-\frac{1}{2\sqrt{6}}$ (b) $\frac{1}{5}$ (c) $-\frac{1}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{6}}$