INDUS RANGERS DEFENCE ACADEMY

4TH Floor Shakumbhari Complex, Chetmani Near: IP Vijaya Mall Bhelupur Varanasi, (U.P.– 221010)

NDA [MATHEMATICS]
PRACTICE SHEET – 5

Time: 2:30 Hr **M. M.: 300**

General Instructions:

- There are 120 questions
- All questions have 2.5 marks for correct and – 0.83 mark for wrong answer as a penalty.
- The name, date, and exam must be filled in OMR sheet otherwise the result will not be published.
- OMR fill should be properly.

1. If matrix $A = \begin{bmatrix} 1-i & i \\ -i & 1-i \end{bmatrix}$ where $i = \sqrt{-1}$, then which one of the following is correct?
   A. A is Hermitian
   B. A is Skew – Hermitian
   C. $(\bar{A})^T + A$ is Hermitian
   D. $(\bar{A})^T + A$ is Skew – Hermitian
2. The term independent of x in the binomial expansion of $\left(\frac{2}{x^2} - \sqrt{x}\right)^{10}$ is equal to
   A. 180 B. 120 C. 90 D. 72
3. If $(1 + 2x - x^2)^6 = a_0 + a_1x + a_2x^2 + \cdots + a_{12}x^{12}$, then what is $a_0 - a_1 + a_2 - a_3 + a_4 - \cdots + a_{12}$ equal to?
   A. 32 B. 64 C. 2048 D. 4096
4. If C(20, n + 2) = C(20, n – 2), then what is n equal to?
   A. 18 B. 25 C. 10 D. 12
5. For how many values of k, is the matrix $\begin{bmatrix} 0 & k & 4 \\ -k & 0 & -5 \\ -k & k & -1 \end{bmatrix}$ singular?
   A. Only one B. Only two C. Only four D. Infinite
6. The number $(1101101 + 1011011)_2$ can be written in decimal system as
   A. $(198)_{10}$ B. $(199)_{10}$ C. $(200)_{10}$ D. $(201)_{10}$
7. What is the value of $\frac{1}{10}\log_5 1024 - \log_5 10 + \frac{1}{5}\log_5 3125$?
   A. 0 B. 1 C. 2 D. 3
8. If $x = \log_c(ab)$, $y = \log_a(ab)$, $z = \log_b(ca)$, then which of the following is correct?
   A. xyz = 1
   B. x + y + z = 1
   C. $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1} = 1$
   D. $(1 + x)^{-2} + (1 + y)^{-2} + (1 + z)^{-2} = 1$
9. Let $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. If AB = C, then what is the value of the determinant of the matrix A?
   A. – 10 B. – 14 C. – 24 D. – 34
10. If $1.5 \leq x \leq 4.5$, then which one of the following is correct?
    A. (2x – 3) (2x – 9) > 0
    B. (2x – 3) (2x – 9) < 0

C. $(2x - 3)(2x - 9) \geq 0$ D. $(2x - 3)(2x - 9) \leq 0$

11. Let S = {1, 2, 3, …}. A relation R on S × S is defined by xRy if $\log_a x > \log_a y$ when a = 1/2. Then the relation is

A. Reflexive only B. Symmetric only

C. Transitive only D. Both symmetric and transitive

12. What is the value of the determinant $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$ where $i = \sqrt{-1}$?

A. 0 B. – 2 C. 4 I D. – 4 i

13. Let $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, then what is AB equal to?

A. $\begin{bmatrix} ax + hy + gz \\ y \\ z \end{bmatrix}$ B. $\begin{bmatrix} ax + hy + gz \\ hx + by + fz \\ z \end{bmatrix}$

C. $\begin{bmatrix} ax + hy + gz \\ hx + by + fz \\ gx + fy + cz \end{bmatrix}$

D. $[ax + hy + gz \quad hx + by + fz \quad gx + fy + cz]$

14. What is the number of ways in which the letters of the word 'ABLE' can be arranged so that the vowels occupy even places?

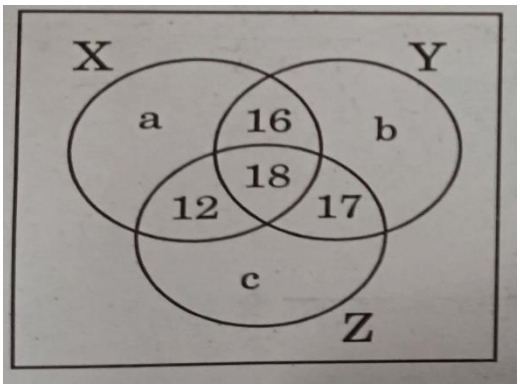
A. 2 B. 4 C. 6 D. 8

15. What is the maximum number of points of intersection of 5 non-overlapping circles?

A. 10 B. 15 C. 20 D. 25

Directions for the following three (03) items:

Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by n(Z) which is equal to 90.



16. If the number of elements in Y and Z are in the ratio 4 : 5, then what is the value of b?

A. 18 B. 19 C. 21 D. 23

17. What is the value of

$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$?

A. a + b + 43 B. a + b + 63
C. a + b + 96 D. a + b + 106

18. If the number of elements belonging to neither X, nor Y, nor Z is equal to p, then what is the number of elements in the complaint of X?
A. p + b + 60 B. p + b + 40
C. p + a + 60 D. p + a + 40

Directions for the following three (02) items:

Read the following information and answer the two items that follow:

Let $\frac{\tan 3A}{\tan A} = K$, where $\tan A \neq 0 \ and \ K \neq \frac{1}{3}$.

19. What is $tan^2 A$ equal to?
A. $\frac{K+3}{3K-1}$ B. $\frac{K-3}{3K-1}$ C. $\frac{3K-3}{K-3}$ D. $\frac{K+3}{3K+1}$
20. For real values of tan A, K cannot lie between
A. $\frac{1}{3} \ and \ 3$ B. $\frac{1}{2} \ and \ 2$ C. $\frac{1}{5} \ and \ 5$ D. $\frac{1}{7} \ and \ 7$

Directions for the following three (02) items:

Read the following information and answer the two items that follow:

ABCD is trapezium such that AB and CD are parallel and BC is perpendicular to them. Let ∠ADB = θ, ∠ABD = α, BC = p and CD = q.

21. Consider the following:
(a) AD sin θ = AB sin α
(b) BD sin θ = AB sin (θ + α)
Which of the above is/are correct?
A. (a) only B. (b) only
C. Both (a) and (b) D. Neither (a) and (b)

22. What is AB equal to?
A. $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ B. $\frac{(p^2-q^2)\cos\theta}{p\cos\theta+q\sin\theta}$ C. $\frac{(p^2+q^2)\sin\theta}{q\cos\theta+p\sin\theta}$ D. $\frac{(p^2+q^2)\cos\theta}{q\cos\theta+p\sin\theta}$
23. If $\tan\theta = \frac{\cos 17^\circ - \sin 17^\circ}{\cos 17^\circ + \sin 17^\circ}$, then what is the value of θ?
A. 0° B. 28° C. 38° D. 52°
24. A and B are positive acute angles such that $\cos 2B = 3\ sin^2 A$ and 3 sin 2A = 2 sin 2B. What is the value of (A + 2B)?
A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$
25. What is
$\sin 3x + \cos 3x + 4\ sin^3 x - 3\sin x + 3\cos x - 4\ cos^3 x$ equal to?
A. 0 B. 1 C. 2 sin 2x D. 4 cos 4x
26. The value of ordinate of the graph of y = 2 + cos x lies in the interval
A. [0, 1] B. [0, 3] C. [– 1, 1] D. [1, 3]

27. What is the value of $8 \cos 10° . \cos 20° . \cos 40°$?

A. $\tan 10°$ B. $\cot 10°$ C. cosec $10°$ D. $\sec 10°$

28. What is the value of $\cos 48° - \cos 12°$?

A. $\frac{\sqrt{5}-1}{4}$ B. $\frac{1-\sqrt{5}}{4}$ C. $\frac{\sqrt{5}+1}{2}$ D. $\frac{1-\sqrt{5}}{8}$

29. Consider the following statements:

(i) If ABC is a right-angled triangle, right-angled at A and if $\sin B = \frac{1}{3}$, then $cosec\ C = 3$

(ii) If b cos B = c cos C and if the triangle ABC is not right-angled, then ABC must be isosceles.

Which of the above statements is/are correct?

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) and nor (ii)

30. Consider the following statements:

(i) If in a triangle ABC, A = 2B and b = c, then it must be an obtuse-angled triangle.

(ii) There exists no triangle ABC with A = 40°, B = 65° and $\frac{a}{c} = \sin 40° \ cosec\ 15°$.

Which of the above statements is/are correct?

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) nor (ii)

Directions for the following three (03) items:

Read the following information and answer the three items that follow:

Let $a\ sin^2 x + b\ cos^2 x = c; b\ sin^2 y + a\ cos^2 y = d\ and\ p \tan x = q \tan y.$

31. What is $tan^2 x$ equal to?

A. $\frac{c-b}{a-c}$ B. $\frac{a-c}{c-b}$ C. $\frac{c-a}{c-b}$ D. $\frac{c-b}{c-a}$

32. What is $\frac{d-a}{b-d}$ equal to?

A. $sin^2 y$ B. $cos^2 y$ C. $tan^2 y$ D. $cot^2 y$

33. What is $\frac{p^2}{q^2}$ equal to?

A. $\frac{(b-c)(b-d)}{(a-d)(a-c)}$ B. $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ C. $\frac{(d-a)(c-a)}{(b-c)(d-b)}$ D. $\frac{(b-c)(b-d)}{(c-a)(a-d)}$

Directions for the following three (03) items:

Read the following information and answer the three items that follow:

Let $t_n = sin^n \theta + cos^n \theta$

34. What is $\frac{t_3 - t_5}{t_5 - t_7}$ equal to?

A. $\frac{t_1}{t_3}$ B. $\frac{t_3}{t_5}$ C. $\frac{t_5}{t_7}$ D. $\frac{t_1}{t_7}$

35. What is $t_1^2 - t_2$ equal to?

A. $\cos 2\theta$ B. $\sin 2\theta$ C. $2 \cos \theta$ D. $2 \sin \theta$

36. What is the value of t_{10} where $\theta = 45°$?

A. 1 B. $\frac{1}{4}$ C. $\frac{1}{16}$ D. $\frac{1}{32}$

Directions for the following three (03) items:

Read the following information and answer the three items that follow:

Let $\alpha = \beta = 15°$

37. What is the value of $\sin\alpha + \cos\beta$?

A. $\frac{1}{\sqrt{2}}$ B. $\frac{1}{2\sqrt{2}}$ C. $\frac{\sqrt{3}}{2\sqrt{2}}$ D. $\frac{\sqrt{3}}{\sqrt{2}}$

38. What is the value of $\sin 7\alpha - \cos 7\beta$?

A. $\frac{1}{\sqrt{2}}$ B. $\frac{1}{2\sqrt{2}}$ C. $\frac{\sqrt{3}}{2\sqrt{2}}$ D. $\frac{\sqrt{3}}{\sqrt{2}}$

39. What is $\sin(\alpha + 1°) + \cos(\beta + 1°)$ equal to?

A. $\sqrt{3}\cos 1° + \sin 1°$ B. $\sqrt{3}\cos 1° - \frac{1}{2}\sin 1°$

C. $\frac{1}{\sqrt{2}}\left(\sqrt{3}\cos 1° - \sin 1°\right)$ D. $\frac{1}{2}\left(\sqrt{3}\cos 1° + \sin 1°\right)$

40. If $\sin x + \sin y = \cos y - \cos x$, where $0 < y < x < \frac{\pi}{2}$, then what is $\tan\left(\frac{x-y}{2}\right)$ equal to?

A. 0 B. $\frac{1}{2}$ C. 1 D. 2

41. If A is a matrix of order 3 × 5 and B is a matrix of order 5 × 3, then the order of AB and BA will respectively be

A. 3 × 3 and 3 × 3 B. 3 × 5 and 5 × 3

C. 3 × 3 and 5 × 5 D. 5 × 3 and 3 × 5

42. If $p^2, q^2\ and\ r^2$ (where p, q, r > 0) are in GP, then which of the following is/are correct?

(i) p, q and r are in GP.

(ii) $\ln p$, $\ln q\ and\ \ln r$ are in A.P.

Select the correct answer using the code given below:

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) and (ii)

43. If $\cot\alpha\ and\ \cot\beta$ are the roots of the equation $x^2 - 3x + 2 = 0$, then what is $\cot(\alpha + \beta)$ equal to?

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. 2 D. 3

44. The roots α and β of a quadratic equation, satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is the number of such quadratic equations?

A. 0 B. 2 C. 3 D. 4

45. What is the argument of the complex number $\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$, where $i = \sqrt{-1}$?

A. 240° B. 210° C. 120° D. 60°

46. What is the modulus of the complex number $\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$, where $i = \sqrt{-1}$?

A. $\frac{1}{2}$ B. 1 C. $\frac{3}{2}$ D. 2

47. Consider the proper subsets of {1, 2, 3, 4}. How many of these proper subsets are superset of the set {3}?
A. 5 B. 6 C. 7 D. 8

48. Let p, q, and r be three distant positive real numbers. If $D = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$, then which one of the following is correct?
A. D < 0 B. D ≤ 0 C. D > 0 D. D ≥ 0

49. Which is the sum of the last five coefficients in the expansion of $(1 + x)^9$ when it is expanded in ascending powers of x?
A. 256 B. 512 C. 1024 D. 2048

50. Consider the following in respect of a non – singular matrix of order 3:
(i) A (adj A) = (adj A) A
(ii) |adj A| = |A|
Which of the above statements is/are correct?
A. (i) only B. (ii) only
C. Both (i) and (ii) D. Neither (i) and (ii)

51. The centre of the circle $(x - 2a)(x - 2b) + (y - 2c)(y - 2d) = 0$ is
A. (2a, 2c) B. (2b, 2d) C. (a + b, c + d) D. (a – b, c – d)

52. The point (1, – 1) is one of the vertices of a square. If 3x + 2y = 5 is the equation of one diagonal of the square, then what is the equation of the other diagonal?
A. 3x – 2y = 5 B. 2x – 3y = 1 C. 2x – 3y = 5 D. 2x + 3y = – 1

53. Let P(x, y) be any point on the ellipse $25x^2 + 16y^2 = 400$. If Q(0, 3) and R(0, – 3) are two points, then what is (PQ + PR) equal to?
A. 12 B. 10 C. 8 D. 6

54. If the circumcentre of the triangle formed by the lines x + 2 = 0, y + 2 = 0 and kx + y +2 = 0 is (– 1, – 1), then what is the value of k?
A. – 1 B. – 2 C. 1 D. 2

55. In the parabola, $y^2 = x$, what is the length of the chord passing through the vertex and inclined to the x–axis at an angle θ?
A. $\sin\theta \, . \, sec^2\theta$ B. $\cos\theta \, . \, cosec^2\theta$
C. $\cot\theta \, . \, sec^2\theta$ D. $2\tan\theta \, . \, cosec^2\theta$

56. Under which condition, are the points (a, b), (c, d) and (a – c, b – d) collinear?
A. ab = cd B. ac = bd C. ad = bc D. abc = d

57. Let ABC be a triangle. If D(2, 5) and E(5, 9) are the mid – points of the sides AB and AC respectively, then what is the length of the side BC?
A. 8 B. 10 C. 12 D. 14

58. If the foot of the perpendicular drawn from the point (0, k) to the line 3x – 4y – 5 = 0 is (3, 1), then what is the value of k?
A. 3 B. 4 C. 5 D. 6

59. What is the obtuse angle between the lines whose slopes are $2-\sqrt{3}$ and $2+\sqrt{3}$?

A. 105° B. 120° C. 135° D. 150°

60. If $3x-4y-5=0$ and $3x-4y+15=0$ are the equations of a pair of opposite sides of a square, then what is the area of the square?

A. 4 square units B. 9 square units

C. 16 square units D. 25 square units

61. What is the length of the diameter of the sphere whose centre is at (1, –2, 3) and which touches the plane 6x – 3y + 2z – 4 = 0?

A. 1 units B. 2 units C. 3 units D. 4 units

62. What is the perpendicular distance from the points (2, 3, 4) to the line $\frac{x-0}{1}=\frac{y-0}{0}=\frac{z-0}{0}$?

A. 6 units B. 5 units C. 3 units D. 2 units

63. If a line has direction ratios <a + b, b + c, c + a> , then what is the sum of the squares of its direction cosines?

A. $(a+b+c)^2$ B. $2\,(a+b+c)$ C. 3 D. 1

64. Into how many compartments do the coordinate planes divide the space?

A. 2 B. 4 C. 8 D. 16

65. What is the equation of the plane which cuts an intercept 5 units on the z-axis and is parallel to xy-plane?

A. x + y = 5 B. z = 5 C. z = 0 D. x + y + z = 5

66. If $\hat{a}$ is a unit vector in the xy-plane making an angle 30° with the positive x-axis, then what is $\hat{a}$ equal to?

A. $\frac{\sqrt{3}\,\hat{\imath}+\hat{\jmath}}{2}$ B. $\frac{\sqrt{3}\,\hat{\imath}-\hat{\jmath}}{2}$ C. $\frac{\hat{\imath}+\sqrt{3}\hat{\jmath}}{2}$ D. $\frac{\hat{\imath}-\sqrt{3}\hat{\jmath}}{2}$

67. Let A be a point in space such that $\left|\overrightarrow{OA}\right|=12$, where O is the origin. If $\overrightarrow{OA}$ is inclined at angles 45° and 60° with x-axis and y-axis respectively, then what is $\overrightarrow{OA}$ equal to?

A. $6\hat{\imath}+6\hat{\jmath}\pm\sqrt{2}\hat{k}$ B. $6\hat{\imath}+6\sqrt{2}\hat{\jmath}\pm6\hat{k}$

C. $6\sqrt{2}\hat{\imath}+6\hat{\jmath}\pm6\hat{k}$ D. $3\sqrt{2}\hat{\imath}+3\hat{\jmath}\pm6\hat{k}$

68. Two adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$. What is the magnitude of dot product of vectors which represent its diagonal?

A. 21 B. 25 C. 31 D. 36

69. If $\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}.\vec{b}\right|^2=144$ and $|\vec{a}|=4$, then what is $\left|\vec{b}\right|$ equal to?

A. 3 B. 4 C. 6 D. 8

70. If the vectors $\vec{a}=2\hat{\imath}-3\hat{\jmath}+\hat{k},\vec{b}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$ and $\vec{c}=\hat{\jmath}+p\hat{k}$ are coplanar, then what is the value of p?

A. 1 B. –1 C. 5 D. –5

71. What is $\lim\limits_{x\to1}\frac{x+x^2+x^3-3}{x-1}$ equal to?

A. 1 B. 2 C. 3 D. 6

72. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

A. 4.4 cm/sec B. 8.4 cm/ sec C. 8.8 cm/sec D. 15.4 cm/sec

73. If $\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, where k ≠ 0, then what is the value of k?

A. $\frac{2}{3}$ B. $\frac{4}{3}$ C. $\frac{8}{3}$ D. 4

74. The order and degree of the differential equation $k\frac{dy}{dx} = \int\left[1+\left(\frac{dy}{dx}\right)^2\right]^{2/3} dx$ are respectively.

A. 1 and 1 B. 2 and 3 C. 2 and 4 D. 1 and 4

75. What is $\lim_{x\to 0}\frac{\sin x \log(1-x)}{x^2}$ equal to?

A. – 1 B. Zero C. – e D. $-\frac{1}{e}$

76. If $f(x) = 3x^2 - 5x + p$ and f(0) and f(1) are opposite in sign, then which of the following is correct?

A. –2 < p < 0 B. – 2 < p < 2 C. 0 < p < 2 D. 3 < p < 5

77. If $e^{\theta\varphi} = c + 4\theta\varphi$, where c is an arbitrary constant and φ is a function of θ, then what is φ dθ equal to?

A. θ dφ B. – θ dφ C. 4θ dφ D. – 4θ dφ

78. If $p(x) = (4e)^{2x}$, then what is $\int p(x)\,dx$ equal to?

A. $\frac{p(x)}{1+2\ln 2} + c$ B. $\frac{p(x)}{2\,(1+2\ln 2)} + c$

C. $\frac{2\,p(x)}{1+\ln 4} + c$ D. $\frac{p(x)}{1+\ln 2} + c$

79. What is the value of $\int_0^{\pi/4}(tan^3x + \tan x)\,dx$?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. 2

80. Let $y = 3x^2 + 2$. If x changes from 10 to 10.1, then what is the total change in y?

A. 4.71 B. 5.23 C. 6.03 D. 8.01

81. If $f(x) = \frac{\sin x}{x}$, where x ∈ R, is to be continuous at x = 0, then the value of the function at x = 0.

A. should be 0 B. should be 1

C. should be 2 D. cannot be determined

82. The solution of the differential equation $dy = (1+y^2)dx$ is

A. $y = \tan x + c$ B. $y = \tan(x+c)$

C. $\tan^{-1}(y+c) = x$ D. $\tan^{-1}(y+c) = 2x$

83. What is $\int\left(e^{\log x} + \sin x\right)\cos x\,dx$ equal to?

A. $\sin x + x\cos x + \frac{sin^2x}{2} + c$ B. $\sin x - x\cos x + \frac{sin^2x}{2} + c$

C. $x\sin x + \cos x + \frac{sin^2x}{2} + c$ D. $x\sin x - x\cos x + \frac{sin^2x}{2} + c$

84. What is the domain of the function $f(x) = \cos^{-1}(x-2)$?

A. [–1, 1] B. [1, 3] C. [0, 5] D. [–2, 1]

85. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight-line y = x?

A. $\frac{1}{2}$ B. 1 C. $\frac{2}{3}$ D. 2

86. If $f(x) = 2x - x^2$, then what is the value of $f(x + 2) + f(x - 2)$ when x = 0?

A. – 8 B. – 4 C. 8 D. 4

87. If $x^m y^n = a^{m+n}$, then what is $\frac{dy}{dx}$ equal to?

A. $\frac{my}{nx}$ B. $-\frac{my}{nx}$ C. $\frac{mx}{ny}$ D. $-\frac{ny}{mx}$

88. What is $\int \frac{dx}{x(x^n+1)}$ equal to?

A. $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right) + c$ B. $\ln\left(\frac{x^n+1}{x^n}\right) + c$

C. $\ln\left(\frac{x^n}{x^n+1}\right) + c$ D. $\frac{1}{n}\ln\left(\frac{x^n+1}{x^n}\right) + c$

89. What is the minimum value of |x – 1|, where x ∈ R?

A. 0 B. 1 C. 2 D. –1

90. What is the value of k such that integration of $\frac{3x^2+8-4k}{x}$ with respect to x, may be a rational function?

A. 0 B. 1 C. 2 D. – 2

91. Consider the following statements for $f(x) = e^{-|x|}$:

(i) The function is continuous at x = 0.

(ii) The function is differentiable at x = 0.

Which of the above statements is/are correct

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) nor (ii)

92. What is the maximum value of $\sin x . \cos x$?

A. 2 B. 1 C. $\frac{1}{2}$ D. $2\sqrt{2}$

93. What is $\lim_{x\to 0} \frac{3^x+3^{-x}-2}{x}$ equal to?

A. 0 B. – 1

C. 1 D. Limit does not exist

94. What is the derivative of $\tan^{-1} x$ with respect to $\cot^{-1} x$?

A. –1 B. 1 C. $\frac{1}{x^2+1}$ D. $\frac{x}{x^2+1}$

95. The function u(x, y) = c which satisfies the differential equation $x(dx - dy) + y(dy - dx) = 0$, is

A. $x^2 + y^2 = xy + c$ B. $x^2 + y^2 = 2xy + c$

C. $x^2 - y^2 = xy + c$ D. $x^2 - y^2 = 2xy + c$

96. What is the minimum value of $3\cos\left(A + \frac{\pi}{3}\right)$ where A ∈ R?

A. – 3 B. – 1 C. 0 D. 3

97. Consider the following statements:

(i) The function $f(x) = \ln x$ increases in the interval (0, ∞).

(ii) The function $f(x) = \tan x$ increases in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Which of the above statements is/are correct?

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) nor (ii)

98. Which one of the following is correct in respect of the graph of $y = \frac{1}{x-1}$?

A. The domain is {x ∈ R | x ≠ 1} and the range is the set of reals.

B. The domain is {x ∈ R | x ≠ 1}, the range is {y ∈ R | y ≠ 0} and the graph intersects y-axis at (0, –1).

C. The domain is the set of reals and the range is the singleton set {0}.

D. The domain is {x ∈ R | x ≠ 1} and the range is the set of points on the y-axis.

99. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) = x$?

A. $y = e^x + c$ B. $y = e^{-x} + c$

C. $y = \ln x + c$ D. $y = 2\ln x + c$

100. Let l be the length and b be the breadth of a rectangle such that $l + b = k$. What is the maximum area of the rectangle?

A. $2k^2$ B. k^2 C. $\frac{k^2}{2}$ D. $\frac{k^2}{4}$

101. The numbers 4 and 9 have frequencies x and (x – 1) respectively. If their arithmetic mean is 6, then what is the value of x?

A. 2 B. 3 C. 4 D. 5

102. If three dice are rolled under the condition that no two dice show the same face, then what is the probability that one of the faces is having the number 6?

A. $\frac{5}{6}$ B. $\frac{5}{9}$ C. $\frac{1}{2}$ D. $\frac{5}{12}$

103. If $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(not\ A) = \frac{1}{2}$, then which one of the following is not correct?

A. $P(B) = \frac{2}{3}$ B. $P(A \cap B) = P(A)P(B)$

C. $P(A \cup B) > P(A) + P(B)$ D. P(not A and not B) = P(not A) P(not B)

104. The sum of deviations of n number of observations measured from 2.5 is 50. The sum of deviations of the same set of observations measured from 3.5 is –50. What is the value of n?

A. 50 B. 60 C. 80 D. 100

105. A data set of n observations has mean 2M, while another data set of 2n observations has mean M. What is the mean of the combined data sets?

A. M B. $\frac{3M}{2}$ C. $\frac{2M}{3}$ D. $\frac{4M}{3}$

Directions for the following three (03) items:

Read the following information and answer the three items that follow:

Marks	Number of Students	
	Physics	Mathematics
10 – 20	8	10
20 – 30	11	21
30 – 40	30	38
40 – 50	26	15
50 – 60	15	10
60 – 70	10	6

106. The difference between number of students under Physics and Mathematics is largest for the interval

A. 20 – 30 B. 30 – 40 C. 40 – 50 D. 50 – 60

107. Consider the following statements:

(i) Modal value of the marks in Physics lies in the interval 30 – 40.

(ii) Median of the marks in Physics is less than that of marks in Mathematics.

Which of the above statements is/are correct?

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) nor (ii)

108. What is the mean of marks in Physics?

A. 38.4 B. 39.4 C. 40.9 D. 41.6

109. What is the standard deviation of the observations $-\sqrt{6}, -\sqrt{5}, -\sqrt{4}, -1, 1, \sqrt{4}, \sqrt{5}, \sqrt{6}$?

A. $\sqrt{2}$ B. 2 C. $2\sqrt{2}$ D. 4

110. If $\sum x_i = 20, \sum x_i^2 = 200$ and n = 10 for an observed variable x, then what is the coefficient of variation?

A. 80 B. 100 C. 150 D. 200

111. What is the probability that February of a leap year selected at random, will have five Sundays?

A. $\frac{1}{5}$ B. $\frac{1}{7}$ C. $\frac{2}{7}$ D. 1

112. The arithmetic mean of 100 observations is 40. Later, it was found that an observation “53” was wrongly read as “83”. What is the correct arithmetic mean?

A. 39.8 B. 39.7 C. 39.6 D. 39.5

113. A husband and wife appear in an interview for two vacancies for the same post. The probability of the husband’s selection is 1/7 and that of the wife’s selection is 1/5. If the events are independent, then the probability of which one of the following is 11/35?

A. At least one of them will be selected B. Only one of them will be selected

C. None of them will be selected D. Both of them will be selected

114. A dealer has a stock of 15 gold coins out of which 6 are counterfeits. A person randomly picks 4 of the 15 gold coins. What is the probability that all the coins picked will be counterfeits?

A. $\frac{1}{91}$ B. $\frac{4}{91}$ C. $\frac{6}{91}$ D. $\frac{15}{91}$

115. A committee of 3 is to be formed from a group of 2 boys and 2 girls. What is the probability that the committee consists of 2 boys and 1 girl?

A. $\frac{2}{3}$ B. $\frac{1}{4}$ C. $\frac{3}{4}$ D. $\frac{1}{2}$

116. In a lottery of 10 tickets numbered 1 to 10, two tickets are drawn simultaneously. What is the probability that both the tickets drawn have prime numbers?

A. $\frac{1}{15}$ B. $\frac{1}{2}$ C. $\frac{2}{15}$ D. $\frac{1}{5}$

117. Let X and Y represent prices (in Rs) of a commodity in Kolkata and Mumbai respectively. It is given that $\bar{X} = 65, \bar{Y} = 67, \sigma_X = 2{\cdot}5, \sigma_Y = 3{\cdot}5$ and r(X, Y) = 0·8. What is the equation of regression of Y on X?

A. Y = 0.175X – 5 B. Y = 1.12X – 5.8

C. Y = 1.12X – 5 D. Y = 0.17X + 5.8

118. Consider a random variable X which follows Binomial distribution with parameters n = 10 and p = $\frac{1}{5}$. Then Y = 10 – X follows Binomial distribution with parameters n and p respectively given by

A. $5, \frac{1}{5}$ B. $5, \frac{2}{5}$ C. $10, \frac{3}{5}$ D. $10, \frac{4}{5}$

119. If A and B are two events such that P(A) = 0.6, P(B) = 0.5 and P(A $\cap$ B) = 0.4, then consider the following statements:

(i) $P(\bar{A} \cup B) = 0.9$

(ii) $P(\bar{B}|\bar{A}) = 0.6$

Which of the above statements is/are correct?

A. (i) only B. (ii) only

C. Both (i) and (ii) D. Neither (i) nor (ii)

120. Three cooks X, Y and Z bake a special kind of cake, and with respective probabilities 0.02, 0.03 and 0.05, it fails to rise. In the restaurant where they work, X bakes 50%, Y bakes 30% and Z bakes 20% of cakes. What is the proportion of failures caused by X?

A. $\frac{9}{29}$ B. $\frac{10}{29}$ C. $\frac{19}{29}$ D. $\frac{28}{29}$